algorithm is not stated to be simply Gauss elimination applied to the matrix problem, and in fact the author states that this "method" has not appeared in textbooks as yet. The iterative methods of Young-Frankel, and Peaceman-Rachford are each discussed twice, (p. 84 and p. 144) and not one of the four definitions is completely accurate. The book is, however, the only existing bridge between *The Elements of Nuclear Reactor Theory* and present computational technique in the reactor field.

R. S. V.

54[S, W].—RONALD A. HOWARD, *Dynamic Programming and Markov Processes*, Technology Press & Wiley, New York, viii + 136 p., 23 cm. Price $5.75.

Consider a physical system S represented at any time t by a state vector $x(t)$. The classical description of the unfolding of the system over time uses an equation of the form $x(t) = F(x(s), s \leqq t)$, where F is a prescribed operation upon the function $x(s)$ for $s \leqq t$. In certain simple cases, this reduces to the usual vector differential equation $dx/dt = g(x)$, $x(0) = c$.

For a variety of reasons, it is sometimes preferable to renounce a deterministic description and to introduce stochastic variables. If we take $x(t)$ to be a vector whose i-th component is now the probability that the system is in state i at time t, and allow only discrete values of time, we can in many cases describe the behavior of the system over time quite simply by means of the equation $x(t + 1) = Ax(t)$. Here $A = (a_{ij})$, $i, j = 1, 2, \cdots, N$, is a transition matrix whose element a_{ij} is the probability that a system in state j at time t will be found in state i at time $t + 1$. Processes of this type are called Markov processes and are fundamental in modern mathematical physics.

So far we have assumed that the observer plays no role in the process. Let us now assume that in some fashion or other the observer has the power to choose the transition matrix A at each stage of the process. We call a process of this type a *Markovian decision process*. It is a special, and quite important, type of dynamic programming process; cf. Chapter XI of R. Bellman, *Dynamic Programming*, Princeton University Press, 1957.

Let us suppose that at any stage of the process, we have a choice of one of a set of matrices, $A(q) = (a_{ij}(q))$. Associated with each choice of q and initial state i is an expected single-stage return $b_i(q)$. We wish to determine a sequence of choices which will maximize the expected return from n stages of the process. Denoting the maximum expected return from an n-stage process by $f_i(n)$, the principle of optimality yields the functional equation

$$f_i(n) = \max_q \left[b_i(q) + \sum_{j=1}^{N} a_{ij}(q) f_j(n - 1)\right].$$

In this form, the determination of optimal policies and the maximum returns is easily accomplished by means of digital computers; see, for example S. Dreyfus, *J. Oper. Soc. of Great Britain*, 1958. Problems leading to similar equations, resolved in similar fashion, arise in the study of equipment replacement and in continuous form in the "optimal inventory" problem; see Chapter Five of the book mentioned above and K. D. Arrow, S. Karlin, and H. Scarf, *Studies in the Mathematical Theory of Inventory and Production*, Stanford University Press, 1959.

As in the case of the ordinary Markov process, a question of great significance is that of determining the asymptotic behavior as $n \to \infty$. It is reasonable to suspect, from the nature of the underlying decision process, that a certain steady-state behavior exists as $n \to \infty$. This can be established in a number of cases.

The author does not discuss these matters at all. This is unfortunate, since there is little value to steady-state analysis unless one shows that the dynamic process asymptotically approaches the steady-state process as the length of the processes increases. Furthermore, it is essential to indicate the rate of approach.

The author sets himself the task of determining steady-state policies under the assumption of their existence. Granted the existence of a "steady state," the functions $f_i(n)$ have the asymptotic form $nc + b_i + o(1)$ as $n \to \infty$, where c is independent of i. The recurrence relations then yield a system of equations for c and the b_i.

This system can be studied by means of linear programming as a number of authors have realized; see, for example, A. S. Manne, "Linear programming and sequential decisions," *Management Science*, vol. 6, 1960, p. 259–268.

Howard uses a different technique based upon the method of successive approximations, in this case an approximation in policy space. It is a very effective technique, as the author shows, by means of a number of interesting examples drawn from questions of the routing of taxicabs, the auto replacement problem, and the managing of a baseball team.

The book is well-written and attractively printed. It is heartily recommended for anyone interested in the fields of operations research, mathematical economics, or in the mathematical theory of Markov processes.

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55[V, X].—GERTRUDE BLANCH, KARL GOTTFRIED GUDERLEY & EMMA MARIAN VALENTINE, *Tables Related to Axial Symmetric Transonic Flow Patterns*, WADC Technical Report 59–710, 1960, Office of Technical Services, U. S. Department of Commerce, Washington 25, D. C., xlviii + 108 p., 27 cm.

The equations of motion of a compressible fluid are non-linear and are generally difficult to handle. In certain cases, such as in the flow past slender bodies of revolution, the equations can be approximated by much simpler ones. For subsonic and supersonic flow these approximating equations are linear. When the flow velocity is nearly equal to one, the approximate equation for the disturbance potential takes the non-linear form

$$-\Phi_x \Phi_{xx} + \Phi_{yy} + \frac{\Phi_y}{y} + \frac{1}{y^2} \Phi_{\omega\omega} = 0$$

when x, y, ω are cylindrical coordinates. K. G. Guderley and his colleague H. Yoshihara have studied the flow past slender bodies at Mach numbers close to one in a series of papers and in a book by Guderley, *Theorie schallnaher Strömungen*, Springer-Verlag, 1957.